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COMMENT

Comment on 'Vibration analysis of fluid-conveying double-walled carbon nanotubes based on nonlocal elastic theory'

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Abstract

Most recently, Lee and Chang (2009 *J. Phys.: Condens. Matter* **21** 115302) combined nonlocal theory and Euler–Bernoulli beam theory in the study of the vibration of the fluid-conveying double-walled carbon nanotube. In this recent published work, the importance of using nonlocal stress tensors consistently has been overlooked, and some ensuring relations were still presented based on the local stress components. Therefore, the governing equations and applied forces obtained in this manner are either inconsistent or incomplete. In this comment, the consistent governing equations for modelling free transverse vibration of the fluid-conveying double-walled carbon nanotube using the nonlocal Euler–Bernoulli beam model are derived.

A mathematical solution to a coupled vibration problem of fluid-conveying double-walled carbon nanotubes (DWCNTs) based on nonlocal Euler–Bernoulli beam theory has been reported recently by Lee and Chang [1], where the governing equation of DWCNTs for conveying fluid by considering the effect of van der Waals interaction between the inner and outer tubes is expressed as

$$EI_1\frac{\partial^4 Y_1}{\partial X^4}+2m_f v\frac{\partial^2 Y_1}{\partial t\partial X}+m_f v^2\frac{\partial^2 Y_1}{\partial X^2}+\frac{\partial^2}{\partial t^2}\left[(m_{c1}+m_f)Y_1-m_{c1}(e_0a)^2\frac{\partial^2 Y_1}{\partial X^2}\right]=c[Y_2-Y_1] \quad (1a)$$

$$EI_2\frac{\partial^4 Y_2}{\partial X^4}+\frac{\partial^2}{\partial t^2}\left[m_{c2}Y_2-m_{c2}(e_0a)^2\frac{\partial^2 Y_2}{\partial X^2}\right]=c[Y_1-Y_2]. \quad (1b)$$

However, in the above equation there are inconsistencies in the handling of the governing equations and applied forces. Using the same methodology presented by Tounsi *et al* [2], the constitutive relations of nonlocal elasticity theory are presented for application in the analysis of double-walled carbon nanotubes conveying fluid when modelled as Euler–Bernoulli beams.

The governing differential equation of motion for the free vibration of the fluid-conveying tube can be expressed as [2–4]

$$\frac{\partial Q}{\partial x}=m_c\frac{\partial^2 w}{\partial t^2}+F_w-p, \quad (2)$$

where EI_1 and EI_2 stand for the bending rigidities of the inner and outer tubes, $Y_1(X,t)$ and $Y_2(X,t)$ are the bending deflections of the inner and outer tubes, m_{c1} and m_{c2} are the per unit length mass of the tubes, and v and m_f are the uniform mean velocity and the per unit length mass of the flow fluid in the DWCNT, respectively. $p(x)$ is the distributed transverse force along axis x. F_w is the force per unit length induced by the plug flow which is given by [2, 4, 5]

$$F_w = m_f\left(2v\frac{\partial^2 Y}{\partial X\partial t} + v^2\frac{\partial^2 Y}{\partial X^2} + \frac{\partial^2 Y}{\partial t^2}\right), \tag{3}$$

Q is the resultant shear force on the cross section, which satisfies the moment equilibrium condition

$$Q = \frac{\partial M}{\partial X}, \tag{4}$$

for the Euler beam, M is the resultant bending moment defined by

$$M = \int_A y\sigma_x \, \mathrm{d}A, \tag{5}$$

where σ_x is the nonlocal axial stress of the nonlocal continuum theory [6].

The one-dimensional nonlocal constitutive relation for the Euler beam can be written as [2, 6–15]

$$\sigma_x - (e_0 a)^2\frac{\partial^2\sigma_x}{\partial X^2} = -Ey\frac{\partial^2 Y}{\partial X^2} \tag{6}$$

where E is the Young's modulus, a is an internal characteristic length (length of C–C bond), and e_0 is a constant for adjusting the model in matching some reliable results by experiments or other models.

According to equation (6), the relation (5) thus can be expressed as

$$M = (e_0 a)^2\frac{\partial^2 M}{\partial X^2} - EI\frac{\partial^2 Y}{\partial X^2} \tag{7}$$

where $I = \int_A y^2\,\mathrm{d}A$ is the moment of inertia.

By substituting equations (2) and (4) into (7), the nonlocal bending moment M and shear force Q can be obtained as

$$M = -EI\frac{\partial^2 Y}{\partial X^2} + (e_0 a)^2\left[m_c\frac{\partial^2 Y}{\partial t^2} + F_w - p\right] \tag{8}$$

and

$$Q = -EI\frac{\partial^3 Y}{\partial X^3} + (e_0 a)^2\left[m_c\frac{\partial^3 Y}{\partial X\partial t^2} + \frac{\partial F_w}{\partial X} - \frac{\partial p}{\partial X}\right]. \tag{9}$$

The equation of motion (2) thus can be expressed by the transverse deflection as

$$p = EI\frac{\partial^4 Y}{\partial X^4} + \rho A\frac{\partial^2 Y}{\partial t^2} + F_w - (e_0 a)^2 \times\left(m_c\frac{\partial^4 Y}{\partial X^2\partial t^2} + \frac{\partial^2 F_w}{\partial X^2} - \frac{\partial^2 p}{\partial X^2}\right). \tag{10}$$

It is known that DWCNTs are distinguished from a traditional elastic beam by their hollow two-layer structures and associated intertube van der Waals forces. As CNTs have high thermal conductivity, it may be regarded that the change of temperature is uniformly distributed in the CNT. Thus equation (10) can be used to each of the inner and outer tubes of the double-walled carbon nanotubes. Assuming that the inner and outer tubes have the same thickness and effective material constants, we have

$$p_{12} = EI_1\frac{\partial^4 Y_1}{\partial X^4} + m_{c1}\frac{\partial^2 Y_1}{\partial t^2} + F_w - (e_0 a)^2 \times\left(m_{c1}\frac{\partial^4 Y_1}{\partial X^2\partial t^2} + \frac{\partial^2 F_w}{\partial X^2} - \frac{\partial^2 p_{12}}{\partial X^2}\right), \tag{11a}$$

$$-p_{12} = EI_2\frac{\partial^4 Y_2}{\partial X^4} + m_{c2}\frac{\partial^2 Y_2}{\partial t^2} - (e_0 a)^2 \times\left(m_{c2}\frac{\partial^4 Y_2}{\partial X^2\partial t^2} + \frac{\partial^2 p_{12}}{\partial X^2}\right), \tag{11b}$$

where subscripts 1 and 2 are used to denote the quantities associated with the inner and outer tubes, respectively, p_{12} denotes the van der Waals pressure per unit axial length exerted on the inner tube by the outer tube.

The deflections of two tubes are coupled through the van der Waals force [16]. The van der Waals interaction potential, as a function of the interlayer spacing between two adjacent tubes, can be estimated by the Lennard-Jones model. The interlayer interaction potential between two adjacent tubes can be simply approximated by the potential obtained for two flat graphite monolayers, denoted by $g(\Delta)$, where Δ is the interlayer spacing [17, 18]. Since the interlayer spacing is equal or very close to an initial equilibrium spacing, the initial van der Waals force is zero for each of the tubes provided they deform coaxially. Thus, for small-amplitude sound waves, the van der Waals pressure should be a linear function of the difference of the deflections of the two adjacent layers at the point as follows:

$$p_{12} = c(Y_2 - Y_1) \tag{12}$$

where c is the intertube interaction coefficient per unit length between two tubes, which can be estimated by [7, 14]

$$c = \frac{320(2R_1)\ \mathrm{erg\ cm^{-2}}}{0.16d^2} \qquad (d = 0.142\ \mathrm{nm}) \tag{13}$$

where R_1 is the radius of the inner tube.

Thus, according to equations (3), (11) and (12), the governing equation of motion for the free vibration of the fluid-conveying tube using nonlocal elastic theory can be expressed as

$$c[Y_2 - Y_1] = EI_1\frac{\partial^4 Y_1}{\partial X^4} + 2m_f v\frac{\partial^2 Y_1}{\partial t\partial X} + m_f v^2\frac{\partial^2 Y_1}{\partial X^2} + (m_{c1} + m_f)\frac{\partial^2 Y_1}{\partial t^2} - (e_0 a)^2\left[(m_{c1} + m_f)\frac{\partial^4 Y_1}{\partial X^2\partial t^2} + 2m_f v\frac{\partial^4 Y_1}{\partial X^3\partial t} + m_f v^2\frac{\partial^4 Y_1}{\partial X^4} - c\left(\frac{\partial^2 Y_2}{\partial X^2} - \frac{\partial^2 Y_1}{\partial X^2}\right)\right], \tag{14a}$$

$$c[Y_1 - Y_2] = EI_2\frac{\partial^4 Y_2}{\partial X^4} + m_{c2}\frac{\partial^2 Y_2}{\partial t^2} - (e_0 a)^2\left[m_{c2}\frac{\partial^4 Y_2}{\partial X^2\partial t^2} - c\left(\frac{\partial^2 Y_1}{\partial X^2} - \frac{\partial^2 Y_2}{\partial X^2}\right)\right], \tag{14b}$$

Equations (8), (9) and (14) are the consistent basic equations of the nonlocal Euler–Bernoulli beam model applied for DWCNTs conveying fluid based on the constitutive relations (6). When $e_0a = 0$, they are reduced to the equations of a classical Euler–Bernoulli beam. It is noted from equation (10) or (14) that the governing equations of the nonlocal beam models derived based on the relations (6) include not only the forces F_w and p themselves but also the term relating their second-order derivatives. It is different from the governing equations of the classical (local) beam models. It was overlooked in Lee and Chang [1], in which the forces F_w and p were not considered in deriving the governing equations and were simply added to the equations (1) as treated for local models, but missing the terms concerning the nonlocal effects. In addition, when $m_f = 0$, the coupled equation (14) can be reduced to the same equation in [7, 11].

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